Flexibility and Optimality of Distillation Column Design

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In economics the cost of production of an industry under changing market conditions over a short period in which capital investment remains fixed, is known as the short run average cost (SRAC). The production cost of the industry over a long period in which capital investment is allowed to adjust to changes in market demand, is known as the long run average cost (LRAC) (Russell et al., 1978). The changes of SRAC with production conditions is, in a sense, a reflection of the flexibility of the process. On the other hand, LRAC is a measurement of the optimality of the process with respect to a specific set of production conditions. The concepts of SRAC and LRAC correspond to the views of costs of a production engineer and a design engineer, respectively. In this work, these concepts are extended to explain the relation between optimality and flexibility considerations in the design of a distillation column.

LRAC and SRAC of a Distillation Column

The LRAC of a distillation column can be defined by:

$$\text{LRAC}\ (Q_o, F_o) = \min_{(EP,\,OP)} C(EP, OP, Q_o, F_o) \qquad (1)$$

The cost of production C, is minimized with respect to both equipment parameters EP (e.g., the actual number of stages and diameter of the column) and operating parameters OP (e.g., the reflux ratio and operating pressure). The optimization is carried out subject to certain production specifications, such as product purities, with a fixed feed rate Q_o and an arbitrary overdesign factor—the flooding factor F_o in this case. This procedure for obtaining the LRAC is exactly the same as in a traditional optimal design (TOD) of a column.

During the operating stage of a column, the equipment parameters are essentially fixed by the original design, but operating parameters can be adjusted to meet a changing processing rate Q. The SRAC is:

$$\text{SRAC}\ (Q_o, F_o, Q) = \min_{(OP)} C[EP(Q_o, F_o), OP, Q] \qquad (2)$$

Beyond the upper and lower operating limits of a column, SRAC can be calculated by imposing penalties:

$$\text{SRAC}\ (Q, Q_o, F_o) = [\text{SRAC}\ (Q_u, Q_o, F_o)*Q_u + PP*(Q - Q_u)]/Q \quad Q > Q_u \qquad (3a)$$

$$= \text{SRAC}\ (Q_l)*Q_l/Q \quad Q < Q_l \qquad (3b)$$

Two example calculations are presented here. In example 1, benzene and toulene are separated. In example 2, *m*- and *o*-xylenes are separated instead. Details for calculating LRAC and SRAC can be found in the supplementary material.

The calculated LRAC's and SRAC's of example 1 are shown in Figure 1. In this range, ($80 \le Q_o \le 120$ kmol/h) the LRAC's decrease with increased throughput Q_o, and increased selected flooding factor F_o (less overdesign). The SRAC's intersect with the LRAC's with the same F_o at the design capacities Q_o. They also exhibit minima at their intersections with the LRAC with $F_o = 100$, that is, the flooding limits of the respective designs. In this example, the selection of larger overdesign factors results in higher LRAC's; in other words, flexibility is traded off with optimality.

LRAC's and SRAC's for example 2 are shown in Figure 2. In this region of relatively small throughput rates ($20 \le Q_o \le 80$ kmol/h), the LRAC's actually decrease as smaller flooding factors are selected. The usual trade-off between flexibility and capital cost does not exist in this zone. The sharpness of SRAC indicates that the column is difficult to operate. This is a result of the combined effect of large changes in efficiency with throughput in a relatively small column, and the difficulty of separating a mixture of close volatility. The behavior of LRAC and SRAC in example 2 becomes similar to that of example 1 as

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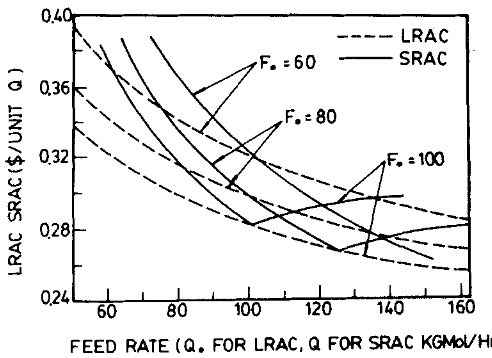

Figure 1. Long and short run average cost curves, example 1.

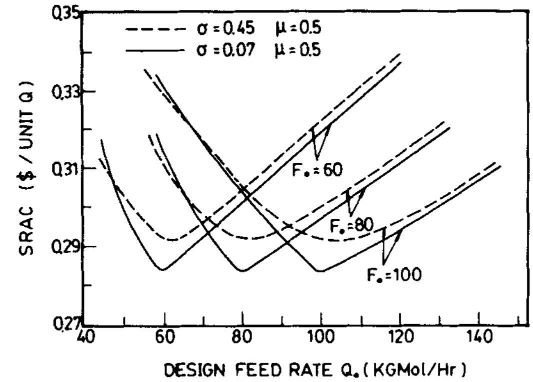

Figure 3. Expected short run average cost curves, example 1.

the throughput rate increases beyond this region; detailed results can be found in the supplementary materials.

Expected Short Run Average Cost and Flexible Optimal Design

Given a throughput rate distribution, the flexibility and the cost of production can be incorporated into a single objective function known as the expected short run average cost ⟨SRAC⟩:

$$\langle \text{SRAC} \rangle (Q_o, F_o) = \int_0^1 P(X)\ \text{SRAC}\ (Q, Q_o, F_o)\ dX \qquad (4)$$

where X is a normalized throughput rate variable: $(Q - Q_{mi})/(Q_{ma} - Q_{mi})$, and ⟨SRAC⟩ accordingly is a function of Q_o and F_o. A flexible optimal design (FOD) is a procedure in which the ⟨SRAC⟩ is minimized to determine the proper design capacity and degree of overdesign.

It is interesting to note that the minimization of ⟨SRAC⟩ corresponds exactly to the two-stage minimization technique proposed by Malik and Hughes (1979) to handle uncertainty in process design. In their inner stage the objective function is a general economic index maximized with respect to certain operating variables; in realistic terms it corresponds to our SRAC. In the outer stage, the objective function is the expected value of another economic index, which is maximized with respect to some design variables, that is, our ⟨SRAC⟩.

Recently, several authors (Morari 1983; Swaney and Grossmann, 1985) have quantified flexibility as a distinctive objective in addition to operating cost, one that can be maximized as operating cost is minimized in a multicriteria search for the best overall design. In this approach, the particular design obtained satisfies all extreme modes of operation. However, it remains questionable whether it is cost-effective to fulfill this constraint in all cases (Kotjabasakis and Linhoff, 1986). In the FOD approach such a criterion can be satisfied by imposing an infinitely high penalty cost when operating limits are exceeded. The assessment of penalty cost function, based on realistic cost estimation and/or engineering experience, allows a cost-effective flexibility factor to be introduced.

Results and Discussion

⟨SRAC⟩ curves of examples 1 and 2 were calculated using beta functions as throughput distributions with standard deviation σ and mean μ (see supplementary material for details). Figure 3 illustrates that for example 1, if flooding factors less than 100% are used, an FOD selection with ⟨SRAC⟩ can be found with a smaller design capacity. It is also found that the design capacity remains unchanged even if the variance of the throughput rate increases, although the ⟨SRAC⟩ increases. In general, for columns with a small number of equilibrium stages, process flexibility is traded off with capital cost. Using cost correlations

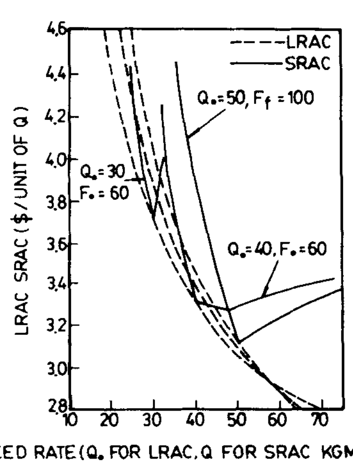

Figure 2. Long and short run average cost curves, example 2.

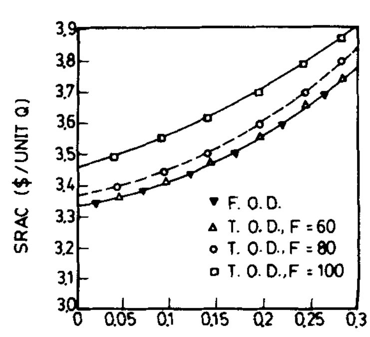

Figure 4. Variation of ⟨SRAC⟩ with σ of a "normal" throughput rate distribution (μ = 0.5), example 2.

employed in this study, we found that such a trade-off is not economically justified. By using the general FOD procedure, an appropriate degree of trade-off can be determined when other processes or cost correlations are considered.

For "normal" throughput rate distributions, ⟨SRAC⟩'s are functions of variances of the distributions only. Figure 4 compares, for example 2, the optimized ⟨SRAC⟩ selected by the FOD and the ⟨SRAC⟩ calculated using TOD's with arbitrary flooding factors and design capacity fixed at the mean. The LRAC's and SRAC's of this example have already demonstrated that the use of smaller flooding factors results in a simultaneous improvement in flexibility and optimality. Therefore, we found that regardless of the value of the variance, a 60% flooding factor is preferred in the FOD approach.

Conclusions

In this study, an alternate approach for characterizing operating flexibility is proposed using the concepts of LRAC and SRAC. An FOD approach is developed that allows inclusion of a cost-effective degree of flexibility by using ⟨SRAC⟩ as the objective function in the optimization procedure. Design of a distillation column using this approach is illustrated.

Our analysis also shows that the trade-off between operating flexibility and capital investment depends on the physics of the process as well as on economic conditions. Moreover, such a trade-off does not justify overdesign for columns with few stages and large operating rates under the cost conditions we have assumed. Furthermore, the trade-off assumption is not valid in the case of a distillation column with a relatively small throughput and a large number of theoretical stages. Use of small flooding factors improves operability and reduces capital cost as well.

Notation

C = cost of production
EP = equipment parameters
F, F_o = flooding factors calculated with operating and design rates
LRAC = long run average cost
$P(X)$ = probability distribution of operating feed rate
PP = penalty for unmet production demand
Q, Q_o = operating, design feed rate
Q_u, Q_l = upper, lower limit of operating feed rate
Q_{ma}, Q_{ml} = maximum, minimum of feed rate distribution
SRAC = short run average cost
⟨SRAC⟩ = expected short run average cost
X = normalized feed rate variable of feed rate distribution
σ, μ = variance, mean of feed rate distribution

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